3)

A floorlet is the counterpart of a put on the floating interest rate and pays the amount max(*rf – rL,*0) at expiry, where we assume that actual floating rate is the underlying spot interest rate, i.e *rL = r* and *rf* is the floor rate. Typically, a floorlet might be purchased by an investor who has to make a stream of payments based on a floating interest rate such as LIBOR, the London InterBank Offer Rate, and who wishes to protect himself against sharp decrease in interest rates. Thus it follows that a floorlet is an insurance against low rates. These interest rate derivatives can be used individually or combined into portfolios: a portfolio of floorlets being known as a floor.

To price interest rate derivatives, it is necessary to model the behaviour of interest rates. It is usual to assume that the spot interest rate *r* obeys the stochastic differential equation,

*dr* = *u*(*r, t*)*dt* + *w*(*r, t*)*dX,*

where *dX* is normally distributed with zero mean and variance *dt* and *w* is the volatility.

Constructing a risk neutral portfolio leads us to the following partial differential equation (PDE) Bond Pricing Equation for the price *V* (*r, t*) of an interest rate derivative,

where*λ*(*r, t*) is the market price of interest rate risk, and *u − λw* is the risk adjusted drift. This equation is valid for times *t ≤ T*, where *T* is the expiry of the derivative. Considering Vasicek model, for which *u−λw* = and *w* = *σ*, with η, γ and *σ* constants rather than functions of time, so that becomes a modified Bond Pricing Equation

with a final condition on zero –coupon bond

*V (r, t; T )* = 1

This model is mean-reverting to a constant level, which is a desirable property for interest rates.

This equation must be solved together with the pay-off at expiry of *V* (*r, T)* = max(*rf − r,* 0) for a floorlet, which lead to modified Vasicek equation for floorlet

Alternatively, we can use a Taylor series solution of the bond pricing equation for short times to expiry.

Substituting this

*Z(r, t; T) = 1 + a(r) (T – t) + b(r) (T – t) 2 + . . .*

into Bond Pricing Equation;

*-a - 2b (T – t) - 3c (T – t) + (w2 + 2(T – t)w) ( (T – t) + (T – t)2  )*

*+ ((u – λw) + (T – t) 2  ) (T – t) ( + (T – t)2  ) – r (1 + a (T – t) + c (T – t)2) + ... = 0*

We find by equating powers of (T – t) that;

*a(r) = -r, b(r) = r2 - r (u – λw) r2 – r (u – λw)*

and

*c(r) = (r2 – r(u – λw)) - (u – λw) (r2 – r(u – λw) - (u – λw) + r2 (r - (u – λw))*

In all of these *u – λw* and *w* are evaluated at *r* and *T*.

From the Taylor series expression for *Z* we find that the yield to maturity is given by

-a + ( *a2- b) (T – t) + (a b – c - a3* ) *(T – t)2 + ...*

After replacing *a, b* and *c* the equation is

-r +  *(u – λw) (T – t) + ...*  as *t T*

Now we can use it to calculate the the cashflow of the floorlet with one month LIBOR floating rate.

We can write this approximately as

The comes from multiplied by the maturity of the one – month rate measure in years ().

After applying Vasicek model, for which *u−λw* = and *w* = *σ, we* got *the final equation*